MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2018/2019

TMA 1111 – MATHEMATICAL TECHNIQUES

(All sections / Groups)

24 OCTOBER 2018
2.30 P.M. – 4.30 P.M.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of **5 pages** only excluding the cover page with **5 Questions** and an **Appendix**.
2. Attempt **ALL FIVE (5)** questions.
3. Please write your answers in the Answer Booklet provided, and **start each question on a new page**.
4. Show all steps.
5. Formulas are provided and attached in Appendix.

Answer ALL FIVE (5) Questions.

Question 1

a. Given that the plane A passes through the points $P(1, 2, 3)$ and $Q(5, 5, 6)$. It is known that the vector $v = \langle 6, 3, 4 \rangle$ is parallel to the plane A.

i. Find the vector $u = \overrightarrow{PQ}$. **[0.5 mark]**

ii. Find the cross product $u \times v$, which is also the vector perpendicular to the plane A. **[1.5 marks]**

iii. Find the equation of the plane A. **[1 mark]**

b. A system of linear equations is given as below.

$$x + 2y + z = 1$$
$$x + 3y + 3z = 1$$
$$3x + y + z = 2$$

i. Write the system of linear equations in matrix form, $AX = B$. **[0.5 mark]**

ii. If Cramer's rule is used to solve the system, what is the value of $|A_3|$? **[1.5 marks]**

iii. Hence, solve the system for x and y if $|A| = 8$, $|A_1| = 5$ and $|A_2| = 2$. **[1 mark]**

c. Given the matrix $A = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$. Find the characteristic equation of A, eigenvalues and eigenvectors for A. **[4 marks]**

Question 2

a. Given the complex numbers $z = 2\text{-}2i$ and $w = \sqrt{2} + \sqrt{2}i$.

i. Write z and w in polar form. **[5 marks]**

ii. Write z and w in exponential form. **[1 mark]**

b. Calculate $\left\{3\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)\right\}^2$. **[2 marks]**

c. Simplify $\dfrac{(\cos 2\theta + i\sin 2\theta)^3(\cos 4\theta + i\sin 4\theta)^5}{\left(\cos\frac{1}{2}\theta + i\sin\frac{1}{2}\theta\right)^2}$. **[2 marks]**

Continued.....

Question 3

a. Find the limit.

i. $\lim_{x\to 1} x^3 - 4x^2 + 6x$. **[1 mark]**

ii. $\lim_{x\to\infty} \frac{2x^3+5}{3x^3-4}$ by using L'Hospital rule or other method. **[1.5 marks]**

b. Find the derivative of the following functions.

i. $y = \frac{7x}{x^2+1}$ by using quotient rule or other method. **[2 marks]**

ii. $y = (4x^3 - 2x)^4$ by using chain rule or other method. **[2 marks]**

c. Integrate

i. $\int_0^1 (x^2 + \sin x + 5)\ dx$. **[1.5 marks]**

ii. $\int x^2 e^{-x} dx$ by using integration by part. **[2 marks]**

Question 4

a. The first order differential equation (ODE) is given below:

$$(9 - y^2)\frac{dy}{dx} = \frac{2e^x}{y}$$

Solve the ODE by using separable method. **[2 marks]**

b. Find the general solution of the following homogeneous linear second order differential equation.

$$y''+3y'-y = 0$$ **[3 marks]**

c. Determine whether the given differential equation, $(5x + 4y)dx + (4x - 8y^3)dy = 0$, is **exact.** Then, solve it. **[5 marks]**

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Question 5

A IT department manager of a company is interested to know the relationship between the number of simultaneous users and the computer terminal respond time (in second). He has collected a random sample of 5 observations with the summary as below.

$$n = 5, \sum x = 15, \sum y = 4.6, \sum x^2 = 55, \sum y^2 = 5.2826, \sum xy = 16.97$$

a. Determine the dependent variable, y. **[0.5 marks]**

If the number of simultaneous users is independent variable, x, and the computer terminal respond time (in second) is dependent variable, y, answer the following questions.

b. Calculate the values of S_{xy}, S_{xx} and S_{yy}. **[4.5 marks]**

c. Find the least square regression line. **[3.5 marks]**

d. Find the correlation coefficient, r. Then, interpret this value. **[1.5 marks]**

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APPENDIX

1. Equation of plane: $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$
2. Characteristics equation: $|\lambda I - A| = 0$
3. Eigenvectors: $(\lambda \mathrm{I} - A)X = 0$
4. Complex numbers:

   $r = \sqrt{x^2+y^2}$

   If x is positive, $\theta = \tan^{-1}\frac{y}{x}$

   If x is negative, $\theta = \tan^{-1}\frac{y}{x} + \pi$

   Polar form: $z = r(\cos\theta + i\sin\theta)$

   Exponential form: $z = re^{i\theta}$

   De Moivre's Formula: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

5. Criterion for an Exact Differential Equation : $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$
6. General solution of the homogeneous linear second order differential equation

   For Distinct real roots: $y = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}$

   For Repeated roots: $y = c_1 e^{\lambda_1 x} + c_2 x e^{\lambda_2 x}$

   For Complex conjugate roots: $y = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$.

7. Derivative Formulas

$\frac{d}{dx}e^x = e^x$
$\frac{d}{dx}\ln x = \frac{1}{x}$
Power Rule: $\frac{d}{dx}x^n = nx^{n-1}$ if $f(x) = x^n$ with $n \in R$
Product Rule: $\frac{d}{dx}(f \cdot g) = f\frac{dg}{dx} + g\frac{df}{dx}$
Quotient Rule: $\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{g\frac{df}{dx} - f\frac{dg}{dx}}{g^2}$ with $g(x) \neq 0$
Chain Rule: If $y = f(u)$ and $u = g(x)$, then $\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$

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8. Integration Formulas

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$, for $n \neq -1$, n rational
$\int \frac{1}{x} dx = \ln x + C$
$\int e^x dx = e^x + C$
$\int \sin kx dx = -\frac{\cos kx}{k} + C$
$\int \cos kx dx = \frac{\sin kx}{k} + C$
Integration by Substitution: If $u = g(x)$ and $du = g'(x)dx$, then $\int f(u)du = \int f(g(x))\frac{d}{dx}g(x)dx$
Integration by Part: $\int udv = uv - \int vdu$

9. $\bar{x} = \frac{\sum x}{n}$

$$S_{xy} = \sum xy - \frac{\left(\sum x \sum y\right)}{n}, \qquad S_{xx} = \sum x^2 - \frac{\left(\sum x\right)^2}{n}, \qquad S_{yy} = \sum y^2 - \frac{\left(\sum y\right)^2}{n}$$

10. $\beta_1 = \frac{S_{xy}}{S_{xx}}$, $\beta_0 = \bar{y} - \beta_1 \bar{x}$

11. Lease square regression line: $\hat{y} = \beta_0 + \beta_1 x$

12. Sample correlation coefficient = $r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}}$

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